Third Semester B.E. Degree Examination, June 2012

Advanced Mathematics – I

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Express $z = \dfrac{2 - \sqrt{3}i}{1 + i}$ in the form a + ib.   **(06 Marks)**

    b. Find modulus and amplitude of $z = \dfrac{3 + i}{2 + i}$.   **(07 Marks)**

    c. Find all the values of $z = \left(\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right)^{3/4}$.   **(07 Marks)**

2 a. Find the nth derivative of $y = e^{ax}\cos(bx + c)$.   **(06 Marks)**

    b. If $y = \sin(m\sin^{-1} x)$ prove that $(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} + (m^2 - n^2)y_n = 0$.   **(07 Marks)**

    c. Expand $y = \log(1 + x)$ in Maclaurins series upto 5th term.   **(07 Marks)**

3 a. If $u = \dfrac{x^2 y^2}{x + y}$, find the value of $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$.   **(06 Marks)**

    b. If $u = 3x^2 + y^2$ and $x^2 - y^2 = 1$, find $\dfrac{du}{dx}$.   **(07 Marks)**

    c. If $x = r\cos\phi$, $y = r\sin\phi$, $z = z$, find $\dfrac{\partial(x, y, z)}{\partial(r, \phi, z)}$.   **(07 Marks)**

4 a. Obtain the reduction formula for $\displaystyle\int_0^{\pi/2} \sin^n x\, dx$ and hence obtain $\displaystyle\int_0^{\pi/2} \sin^4 x\, dx$.   **(06 Marks)**

    b. Evaluate $\displaystyle\int_0^1 x^2(1 - x^2)^{7/2}dx$.   **(07 Marks)**

    c. Evaluate $\displaystyle\int_0^1\int_0^3 x^3 y^3 dx\, dy$.   **(07 Marks)**

5 a. Evaluate $\displaystyle\int_0^1\int_0^2\int_0^3 (x + y + z)dz\, dy\, dx$.   **(06 Marks)**

    b. Evaluate $\displaystyle\int_0^\infty x^2 e^{-4x}dx$ using gamma function.   **(07 Marks)**

    c. Find $\beta\left(\dfrac{5}{2}, \dfrac{3}{2}\right)$ in terms of gamma function..   **(07 Marks)**

6 a. Solve the equation $\sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = 0$. **(06 Marks)**

b. Solve $\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$. **(07 Marks)**

c. Solve $\dfrac{dy}{dx} = (x+y)^2$. **(07 Marks)**

7 a. Solve $\dfrac{dy}{dx} = \dfrac{\sin 2x - \tan y}{x\sec^2 y}$. **(06 Marks)**

b. Solve $\dfrac{d^2y}{dx^2} + x^2 y = x^2$. **(07 Marks)**

c. Solve $\dfrac{dy}{dx} + \sin xy = \sin x \cos x$. **(07 Marks)**

8 a. Solve $(D^2 + a^2)y = x^2$. **(06 Marks)**
b. Solve $(D^3 + D^2 - D - 1)y = e^{2x}$. **(07 Marks)**
c. Solve $(D^4 - 1)y = \sin x + 2$. **(07 Marks)**

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